PARTICLE DATA GROUP NOTES **PDG-19-01**

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On the low-energy behavior of dE/dx

Don Groom

The Bethe equation can be written as

$$-\left\langle \frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] \tag{1}$$

where the variables are defined in RPP2018, in Table 33.1 of section **"33. Passage of particles through matter."**

It is well known that at the lowest energies of applicability dE/dx behaves as $1/\beta^2$, with terms inside the square brackets initially changing very little.

This is NOT TRUE, as can be seen from plots of $\beta^2 dE/dx$ vs T (Figs. 1–3). We find that the initial behavior goes as $1/\beta^\alpha$ rather than as $1/\beta^2$, where α is in the range 1.4–1.7, depending on the incident particle and the absorber. The values decrease somewhat with increasing Z. In Figs. 1–3, NIST data are shown in red and PDG data in black.

We have compared these functions using tables from
`physics.nist.gov/PhysRefData/Star/Text/PSTAR.html`
and `pdg.lbl.gov/AtomicNuclearProperties`. The results are shown in Figs. 1–3, where NIST data are shown in red and PDG data in black. In the regions of interest (5–100 MeV for muons , 10–100 MeV for protons) the stopping powers for Cu from the two sources are nearly congruent. For carbon, the PDG values for muons are about 0.04 MeV cm^2/g higher in the lower part of the range. For uranium the differences are more serious: The PDG values are about 0.14 cm^2/g below the NIST values at 5–6 MeV (muons), then gradually approach agreement above 20 MeV. In any case, we take the NIST values as the gold standard.

In all 6 cases, α was adjusted until $1/\beta^\alpha dE/dx$ "looked" flat for 5–100 MeV for muons and 10–100 MeV for protons. The results are shown in the figures and summarized in Table 1. The exponent α tended to be higher for muons than protons and the values each increased with Z. Given that $1/\beta^\alpha dE/dx$ was not exactly flat, we estimate the error in the exponents at 0.02 or less.

The most surprising feature was that the $1/\beta^\alpha$ behavior described dE/dx to within a few % up to 100 MeV in all cases. Protons in Cu above $\sim$70 MeV is a little worse.

There is no profound physics in this; just the observation that the widely-accepted $1/\beta^2$ behavior is *not even close.*

Absorber	proton	muon
Carbon	1.68	1.72
Copper	1.57	1.62
Uranium	1.44	1.58

Table 1: Eyeball fits for α in region 5–100 MeV for protons and muons incident on amorphous carbon, copper, and uranium. Error is estimated as $\pm$0.02.

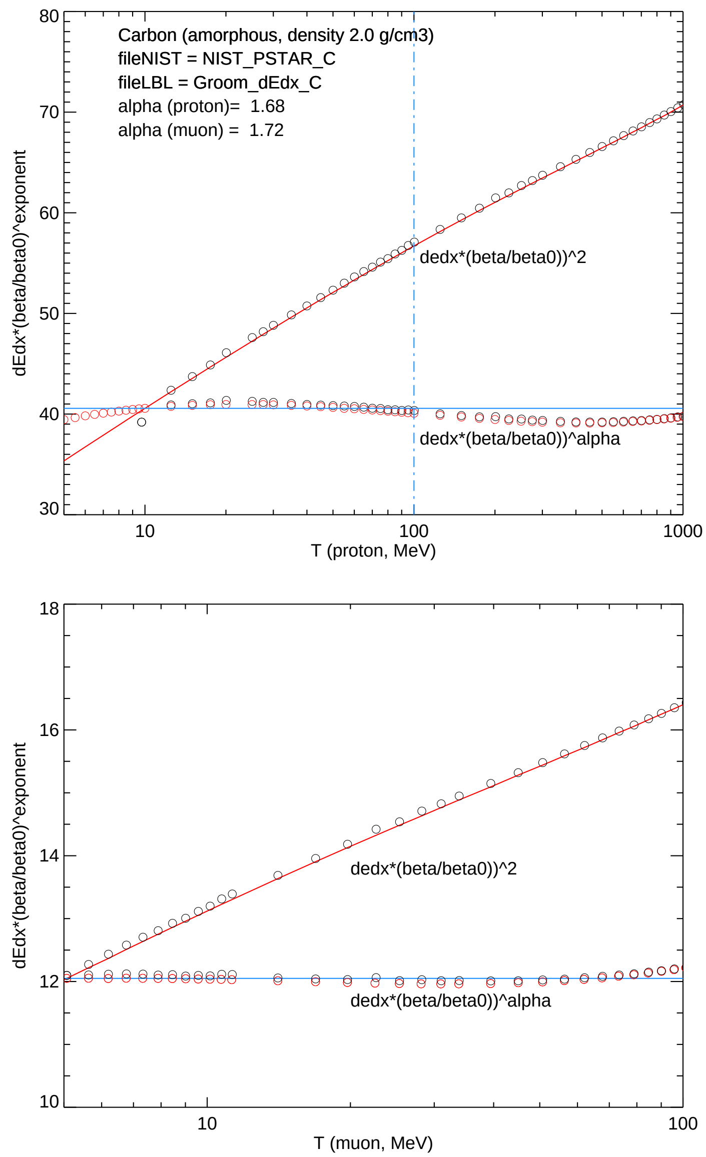

Figure 1: Comparison of $\beta^2 dEdx$ with $\beta^\alpha dE/dx$ in amorphous carbon for incident protons (top) and muons (bottom. Red denotes data from NIST PSTAR, and black indicates data from PDG tables.

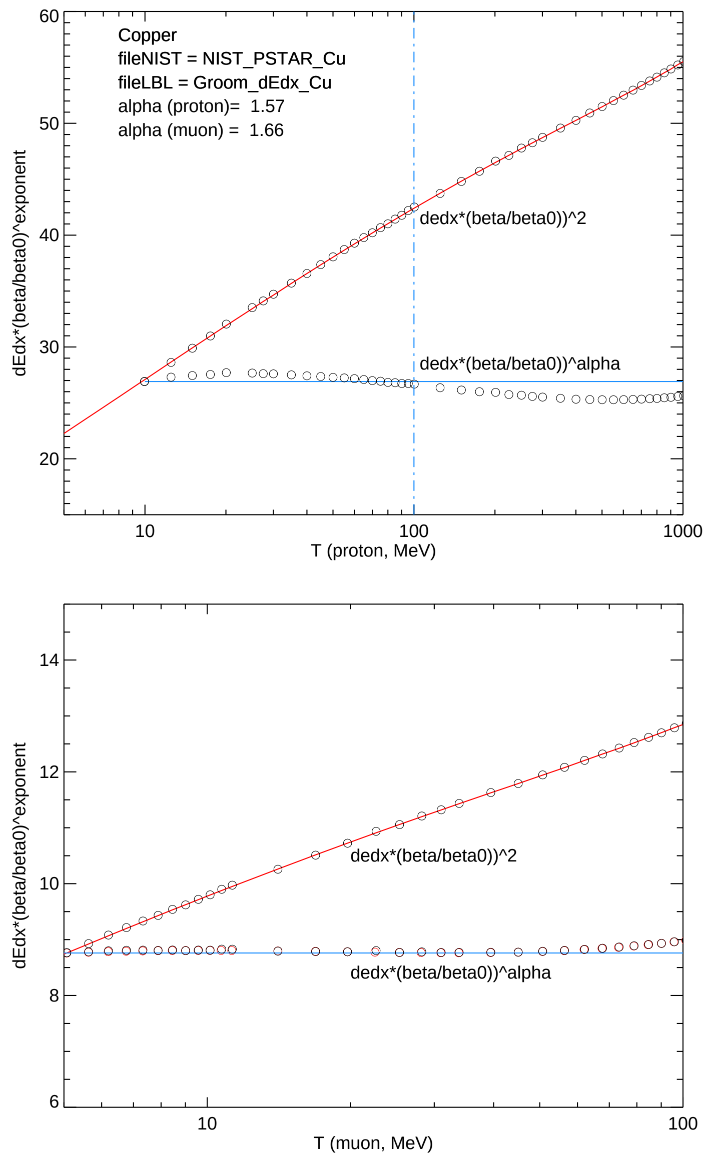

Figure 2: Comparison of $\beta^2 dEdx$ with $\beta^\alpha dE/dx$ in copper for incident protons (top) and muons (bottom). Red denotes data from NIST PSTAR, and black indicates data from PDG tables.

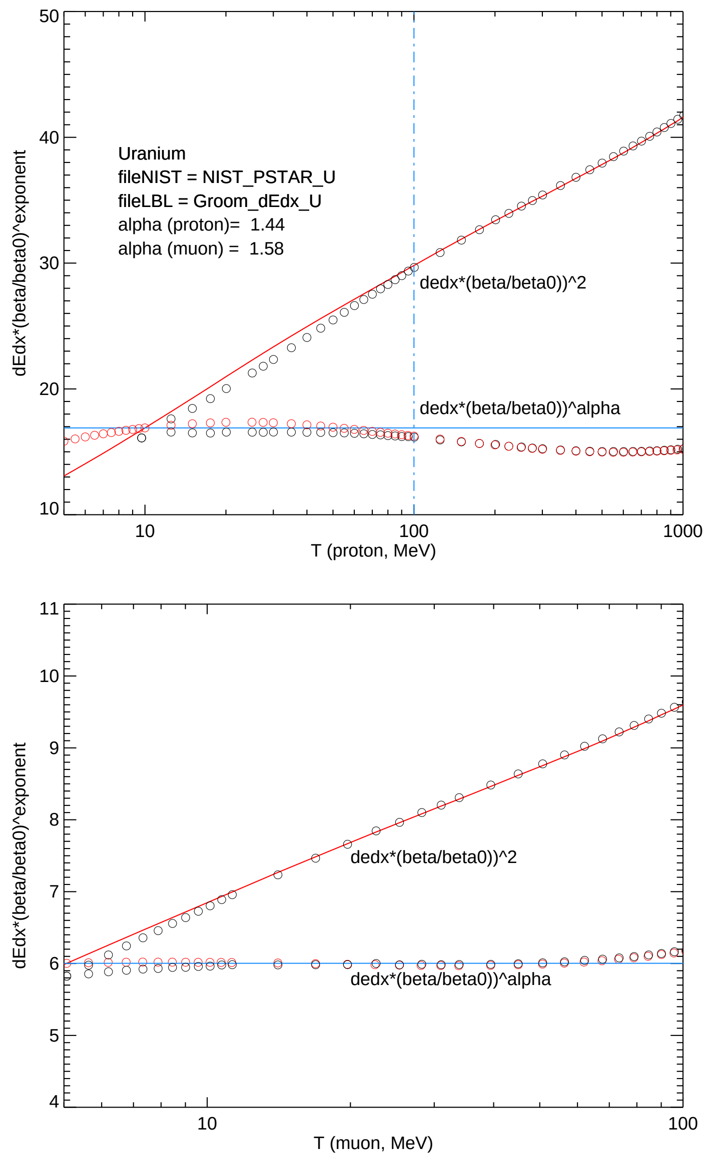

Figure 3: Comparison of $\beta^2 dEdx$ with $\beta^\alpha dE/dx$ in uranium for incident protons (top) and muons (bottom). Red denotes data from NIST PSTAR, and black indicates data from PDG tables.